Stochastic Model Predictive Control for Gust Alleviation during Aircraft Carrier Landing

Gaurav Misra and Xiaoli Bai

Department of Mechanical and Aerospace Engineering
Rutgers, The State University of New Jersey

IEEE American Control Conference
Milwaukee, WI
06/27/2018

Outline

- Motivation
- Stochastic MPC Formulation
- Aircraft and Gust Modeling
- Numerical Results
- Conclusions and Future Work

Motivation

- Aircraft carrier landing challenges
  - Atmospheric turbulence
  - Carrier airwakes
  - Carrier motion
- Requirement: Real-time optimal feedback control
- Previous research: ℓ_1 adaptive control (Ramesh and Subbarao, 2016), nominal MPC (Ngo and Sultan, 2015), dynamic inversion (Denison, 2007)
- Stochastic nature of gusts and airwakes $\rightarrow$ stochastic optimal control

Stochastic MPC

- Optimization based control for offset recovery due to gust

$$\begin{aligned} \text{minimize} \quad & \mathbb{E}[\sum_{k=0}^{N-1}(x_k^T Q x_k + u_k^T R u_k) + x_N^T Q_N x_N] \\ \text{subject to} \quad & x_{k+1} = \bar{A}_d x_k + \bar{B}_d u_k + \bar{E}_d \eta_k \\ & x_k \in \mathbb{X} \\ & u_k \in \mathbb{U} \end{aligned}$$

- Hard polytopic state and control constraints relaxed to individual chance constraints

Stochastic MPC

- In compact form

$$\mathbf{x} = \mathbf{A}x_0 + \mathbf{B}\mathbf{u} + \mathbf{E}\eta$$

- Optimal control problem with probabilistic constraints

$$\begin{aligned} \text{minimize} \quad & \mathbb{E}[\mathbf{x}^\top \mathbf{Q}\mathbf{x} + \mathbf{u}^\top \mathbf{R}\mathbf{u}] \\ \text{subject to} \quad & \mathbb{P}[\mathbf{x} \in \bar{\mathbb{X}}] \geq 1 - \alpha \\ & \mathbb{P}[\mathbf{u} \in \bar{\mathbb{U}}] \geq 1 - \beta \end{aligned}$$

- Adjust α, β for trade-off between conservatism and performance.
- Intractable with non-convex probabilistic constraints

Stochastic MPC

- Assume full state feedback, reconstruct past noise from state and control input
- Affine disturbance feedback policy

$$u_k = \sum_{i=0}^{k-1} G_{k,i}\eta_k + s_k$$

- Compact form

$$\mathbf{u} = \mathbf{G}\boldsymbol{\eta} + \mathbf{s}$$

- Suboptimal but tractable; Origin is ISS w.r.t disturbance input under mild assumptions (Goulart & Kerrigan, 2008)

Stochastic MPC

- Infinite dimensional problem $\rightarrow$ Finite dimensional
- $\eta \sim \mathcal{N}(0, \Sigma)$, individual chance constraints $\rightarrow$ second order cone constraints

$$\Phi^{-1}(1-\alpha_i)\left\|\bar{H}_{x_i}\mathbf{G}+\mathbf{E}\right\|_2 \leq p_i - \bar{H}_{x_i}(\mathbf{A}X_0+\mathbf{Bs})$$
$$\Phi^{-1}(1-\beta_j)\left\|\bar{H}_{u_j}\mathbf{G}\right\|_2 \leq l_j - \bar{H}_{u_j}\mathbf{s}$$

- Constraint set
  - $\mathbb{X} = \{\mathbf{H}_x\mathbf{x} \leq \mathbf{p}\}$ with $\mathbf{H}_x = \mathrm{diag}(H_x, \ldots H_x)$
  - $\mathbb{U} = \{\mathbf{H}_u\mathbf{u} \leq \mathbf{l}\}$ with $\mathbf{H}_u = \mathrm{diag}(H_u, \ldots H_u)$
  - $\mathbf{l} = [l^{\mathsf{T}}, \ldots, l^{\mathsf{T}}]^{\mathsf{T}}$, $\mathbf{p} = [p^{\mathsf{T}}, \ldots, p^{\mathsf{T}}]^{\mathsf{T}}$

Stochastic MPC

Second order cone program formulation of SMPC

$$\begin{aligned} \text{minimize} \quad & b^T \mathbf{s} + \mathbf{tr}(\mathrm{M}_2 \mathbf{G}\boldsymbol{\Sigma} + \mathbf{G}^T \mathrm{M}_1 \mathbf{G}\boldsymbol{\Sigma}) + \mathbf{s}^T \mathrm{M}_1 \mathbf{s} \\ \text{subject to} \quad & \Phi^{-1}(1-\alpha_i) \left\| \bar{H}_{x_i} \mathbf{G} + \mathbf{E} \right\|_2 \leq k_1 \\ & \Phi^{-1}(1-\beta_j) \left\| \bar{H}_{u_j} \mathbf{G} \right\|_2 \leq k_2 \end{aligned}$$

- where
  - $k_1 = p_i - \bar{H}_{x_i}(\mathbf{A}X_0 + \mathbf{B}\mathbf{s})$
  - $k_2 = l_j - \bar{H}_{u_j}\mathbf{s}$
  - $b^T = 2(\mathbf{A}x_0)^T \mathbf{QB}$, $\mathrm{M}_1 = \mathbf{B}^T\mathbf{QB} + \mathbf{R}$ and $\mathrm{M}_2 = 2\mathbf{E}^T\mathbf{QB}$

Aircraft motion

- Linear longitudinal dynamics with gust

$$\begin{bmatrix} \Delta\dot{u} \\ \Delta\dot{w} \\ \Delta\dot{q} \\ \Delta\dot{\theta} \end{bmatrix} = \begin{bmatrix} X_u & X_w & -u_0 \sin\theta_0 & -g\cos\theta_0 \\ Z_u & Z_w & u_0\cos\theta_0 & -g\sin\theta_0 \\ M_u & M_w & M_q & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta u \\ \Delta w \\ \Delta q \\ \Delta\theta \end{bmatrix}$$

$$+ \begin{bmatrix} X_\delta & X_{\delta T} \\ Z_d & Z_{\delta T} \\ M_\delta & M_{\delta T} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\delta_e \\ \Delta\delta_T \end{bmatrix} + \begin{bmatrix} -X_u & -X_w & 0 \\ -Z_u & -Z_w & 0 \\ -M_u & -M_w & -M_q \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_g \\ w_g \\ q_g \end{bmatrix}$$

Aircraft motion

- Aerodynamic coefficients based on the F/A-18 High angle of attack (HARV) model.
- Landing configuration with nominal speed 134 knots and sea level altitude
- Aerodynamic model
  - Leading and trailing edge flaps completely down to 17.6 degrees and 45 degrees
  - Both left and right ailerons down to 42 deg
  - Longitudinal aerodynamics actuator dependency only on elevator deflection

Aircraft motion

- Assuming steady-state descent flight
  - $u_{trim} = 223.1$ ft/s
  - $w_{trim} = 28.4$ ft/s
  - $q_{trim} = 0$ deg/s
  - $\theta_{trim} = 3.72$ deg
- Corresponds to a trim AOA of 7.26 deg and -3.5 deg glideslope
- Trimmed controls
  - $\delta_e = 11.36$ deg
  - $\delta_T = 0.29$

Gust modeling

- Only continuous gusts studied
- Spatially varying stochastic processes with Gaussian distribution
- Dryden form given as

$$\Phi_{u_g}(\Omega) = \sigma_u^2 \frac{L_u}{\pi} \frac{1}{1+(L_u\Omega)^2}$$

$$\Phi_{w_g}(\Omega) = \sigma_w^2 \frac{L_w}{\pi} {\frac{1+3(L_w\Omega)^2}{(1+(L_w\Omega)^2)}}^{2}$$

$$\Phi_{q_g}(\Omega) = \frac{\Omega^2}{1+(\frac{4b\Omega}{\pi})^2}\Phi_{w_g}(\Omega)$$

Gust modeling

- For low altitude ($\sim$ 200 ft)

$$L_w = 100 \ \text{ft} \qquad L_u = \frac{h}{(0.177 + 0.000823h)^{1.2}} \ \text{ft}$$

$$\sigma_w = 0.1 W_{20} \ \text{ft/s} \quad \sigma_u = \frac{\sigma_w}{(0.177 + 0.000823h)^{0.4}} \ \text{ft/s}$$

- Spectral factorization $\rightarrow$ transfer function $\rightarrow$ linear filter driven by white noise

$$\dot{\xi}_w = A_w \xi_w + E_w \eta$$
$$d = C_w \xi_w$$

Gust modeling

- Significance of rotary gust q_g if $\sqrt{\frac{\pi b}{16 L_w}} C_{m_q} > C_{m_\alpha}$
- Augmenting linearized aircraft model with wind dynamics

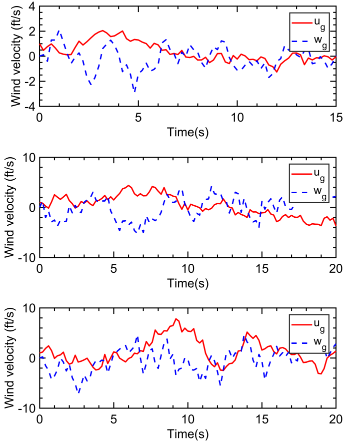
$$\dot{x} = \begin{bmatrix} \dot{x}_l \\ \dot{\xi}_w \end{bmatrix} = \bar{A}x + \bar{B}u + \bar{E}\eta$$

- Discretized version

$$x_{k+1} = \bar{A}_d x_k + \bar{B}_d u_k + \bar{E}_d \eta_k, \quad k \in \mathbb{N}_0$$

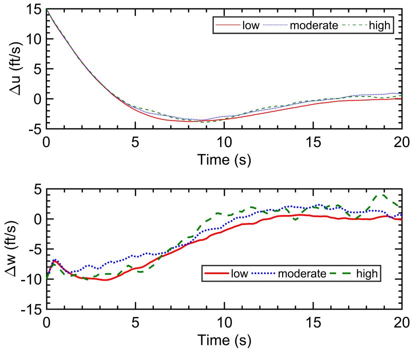
Gust modeling

- Wind gust at low, moderate, and high turbulence

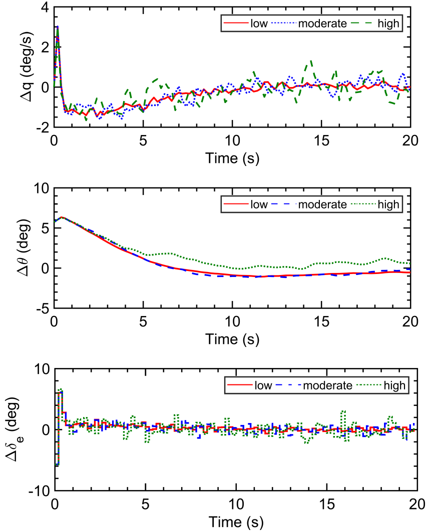


Simulation results

- Perturbed flight with initial state $x = \begin{bmatrix} 15 & -10 & 0 & 0.1 \end{bmatrix}^T$.
- Prediction horizon $N_p = 10$ s, Total time 20 s.

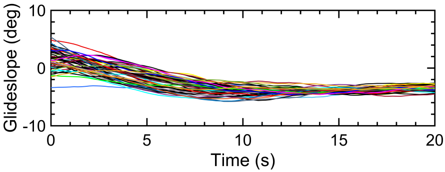


Simulation results

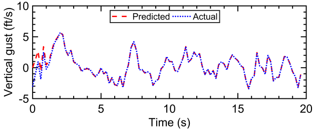


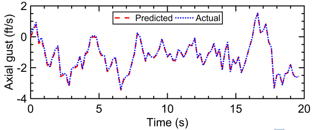
Simulation Results

- Randomized initial conditions



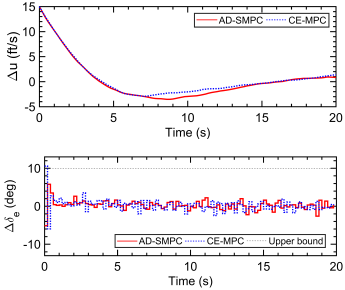
- Noise/wind reconstruction





Numerical Results

- Comparison with certainty equivalent MPC



- Cost comparison

Method	Cost
AD-SMPC	3.23×10^6
CE-MPC	3.47×10^6

Conclusions and Future Work

Summary

- Stochastic MPC for aircraft glideslope recovery in gust
- Chance constrained affine-disturbance feedback MPC formulation
- Tractable, cost efficient solution compared to certainty equivalent MPC

Future directions

- Incomplete state information and measurement noise
- Inclusion of carrier burble components