Stochastic Model Predictive Control for Gust Alleviation during Aircraft Carrier Landing

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IEEE American Control Conference
Milwaukee, WI
06/27/2018
Outline

- Motivation
- Stochastic MPC Formulation
- Aircraft and Gust Modeling
- Numerical Results
- Conclusions and Future Work
Motivation

- Aircraft carrier landing challenges
  - Atmospheric turbulence
  - Carrier airwakes
  - Carrier motion

- Requirement: Real-time optimal feedback control

- Previous research: $\ell_1$ adaptive control (Ramesh and Subbarao, 2016), nominal MPC (Ngo and Sultan, 2015), dynamic inversion (Denison, 2007)

- Stochastic nature of gusts and airwakes $\rightarrow$ stochastic optimal control
Stochastic MPC

- Optimization based control for offset recovery due to gust

\[
\begin{align*}
\text{minimize} & \quad \mathbb{E}\left[ \sum_{k=0}^{N-1} (x_k^T Q x_k + u_k^T R u_k) + x_N^T Q_N x_N \right] \\
\text{subject to} & \quad x_{k+1} = \bar{A}_d x_k + \bar{B}_d u_k + \bar{E}_d \eta_k \\
& \quad x_k \in \mathcal{X} \\
& \quad u_k \in \mathcal{U}
\end{align*}
\]

- Hard polytopic state and control constraints relaxed to individual chance constraints
Stochastic MPC

- In compact form

\[ x = A x_0 + B u + E \eta \]

- Optimal control problem with probabilistic constraints

\[
\begin{align*}
\text{minimize} & \quad E[x^T Q x + u^T R u] \\
\text{subject to} & \quad P[x \in \bar{X}] \geq 1 - \alpha \\
& \quad P[u \in \bar{U}] \geq 1 - \beta
\end{align*}
\]

- Adjust \( \alpha, \beta \) for trade-off between conservatism and performance.

- Intractable with non-convex probabilistic constraints
Stochastic MPC

- Assume full state feedback, reconstruct past noise from state and control input
- Affine disturbance feedback policy
  \[ u_k = \sum_{i=0}^{k-1} G_{k,i} \eta_k + s_k \]
- Compact form
  \[ u = G\eta + s \]
- Suboptimal but tractable; Origin is ISS w.r.t disturbance input under mild assumptions (Goulart & Kerrigan, 2008)
Stochastic MPC

- Infinite dimensional problem $\rightarrow$ Finite dimensional
- $\eta \sim \mathcal{N}(0, \Sigma)$, individual chance constraints $\rightarrow$ second order cone constraints

$$
\Phi^{-1}(1 - \alpha_i) \| \tilde{H}_{x_i} \mathbf{G} + \mathbf{E} \|_2 \leq p_i - \tilde{H}_{x_i} (\mathbf{A} \mathbf{X}_0 + \mathbf{B} \mathbf{s}) \\
\Phi^{-1}(1 - \beta_j) \| \tilde{H}_{u_j} \mathbf{G} \|_2 \leq l_j - \tilde{H}_{u_j} \mathbf{s}
$$

- Constraint set

  - $\mathbf{X} = \{ \mathbf{H}_x \mathbf{x} \leq \mathbf{p} \}$ with $\mathbf{H}_x = \text{diag}(H_x, \ldots H_x)$
  - $\mathbf{U} = \{ \mathbf{H}_u \mathbf{u} \leq \mathbf{l} \}$ with $\mathbf{H}_u = \text{diag}(H_u, \ldots H_u)$
  - $\mathbf{l} = [l^T, \ldots, l^T]^T$, $\mathbf{p} = [p^T, \ldots, p^T]^T$
Stochastic MPC

Second order cone program formulation of SMPC

\[
\begin{align*}
\text{minimize} & \quad b^T s + \text{tr}(M_2 G\Sigma + G^T M_1 G\Sigma) + s^T M_1 s \\
\text{subject to} & \quad \Phi^{-1}(1 - \alpha_i) \| \tilde{H}_{x_i} G + E \|_2 \leq k_1 \\
& \quad \Phi^{-1}(1 - \beta_j) \| \tilde{H}_{u_j} G \|_2 \leq k_2
\end{align*}
\]

where

\[
\begin{align*}
& k_1 = p_i - \tilde{H}_{x_i} (A\tilde{X}_0 + B s) \\
& k_2 = l_j - \tilde{H}_{u_j} s \\
& b^T = 2(A\tilde{X}_0)^T Q B, \quad M_1 = B^T Q B + R \quad \text{and} \quad M_2 = 2E^T Q B
\end{align*}
\]
### Aircraft motion

#### Linear longitudinal dynamics with gust

\[
\begin{bmatrix}
\Delta \dot{u} \\
\Delta \dot{w} \\
\Delta \dot{q} \\
\Delta \dot{\theta}
\end{bmatrix} =
\begin{bmatrix}
X_u & X_w & -u_0 \sin \theta_0 & -g \cos \theta_0 \\
Z_u & Z_w & u_0 \cos \theta_0 & -g \sin \theta_0 \\
M_u & M_w & M_q & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
\Delta u \\
\Delta w \\
\Delta q \\
\Delta \theta
\end{bmatrix}
\]

\[
+ \begin{bmatrix}
X_\delta & X_\delta T \\
Z_\delta & Z_\delta T \\
M_\delta & M_\delta T \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
\Delta \delta e \\
\Delta \delta T
\end{bmatrix}
+ \begin{bmatrix}
-X_u & -X_w & 0 \\
-Z_u & -Z_w & 0 \\
-M_u & -M_w & -M_q \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
u_g \\
w_g \\
q_g
\end{bmatrix}
\]
Aircraft motion

- Aerodynamic coefficients based on the F/A-18 High angle of attack (HARV) model.
- Landing configuration with nominal speed 134 knots and sea level altitude
- Aerodynamic model
  - Leading and trailing edge flaps completely down to 17.6 degrees and 45 degrees
  - Both left and right ailerons down to 42 deg
  - Longitudinal aerodynamics actuator dependency only on elevator deflection
Aircraft motion

- Assuming steady-state descent flight
  - $u_{trim} = 223.1$ ft/s
  - $w_{trim} = 28.4$ ft/s
  - $q_{trim} = 0$ deg/s
  - $\theta_{trim} = 3.72$ deg
- Corresponds to a trim AOA of $7.26$ deg and $-3.5$ deg glideslope
- Trimmed controls
  - $\delta_e = 11.36$ deg
  - $\delta_T = 0.29$
Gust modeling

- Only continuous gusts studied
- Spatially varying stochastic processes with Gaussian distribution
- Dryden form given as

\[
\Phi_{ug}(\Omega) = \sigma_u^2 \frac{L_u}{\pi} \frac{1}{1 + (L_u \Omega)^2} \\
\Phi_{wg}(\Omega) = \sigma_w^2 \frac{L_w}{\pi} \frac{1 + 3(L_w \Omega)^2}{(1 + (L_w \Omega)^2)^2} \\
\Phi_{qg}(\Omega) = \frac{\Omega^2}{1 + \left(\frac{4b\Omega}{\pi}\right)^2} \Phi_{wg}(\Omega)
\]
Gust modeling

- For low altitude (∼ 200 ft)

\[
L_w = 100 \text{ ft} \quad L_u = \frac{h}{(0.177 + 0.000823h)^{1.2}} \text{ ft}
\]

\[
\sigma_w = 0.1W_{20} \text{ ft/s} \quad \sigma_u = \frac{\sigma_w}{(0.177 + 0.000823h)^{0.4}} \text{ ft/s}
\]

- Spectral factorization \(\rightarrow\) transfer function \(\rightarrow\) linear filter driven by white noise

\[
\dot{\xi}_w = A_w\xi_w + E_w\eta
\]

\[
d = C_w\xi_w
\]
Gust modeling

- Significance of rotary gust $q_g$ if $\sqrt{\frac{\pi b}{16L_w}} C_{m_q} > C_{m\alpha}$

- Augmenting linearized aircraft model with wind dynamics

\[
\dot{x} = \begin{bmatrix} \dot{x}_l \\ \dot{\xi}_w \end{bmatrix} = \bar{A}x + \bar{B}u + \bar{E}\eta
\]

- Discretized version

\[
x_{k+1} = \bar{A}_d x_k + \bar{B}_d u_k + \bar{E}_d \eta_k, \quad k \in \mathbb{N}_0
\]
Gust modeling

- Wind gust at low, moderate, and high turbulence

![Wind velocity graphs](image-url)
Simulation results

- Perturbed flight with initial state
  \(x = \begin{bmatrix} 15 & -10 & 0 & 0.1 \end{bmatrix}^T\).
- Prediction horizon \(N_p = 10\) s, Total time 20 s.
Simulation results

![Simulation results](chart1)

**Simulation results**

- **Δq (deg/s)**
  - Low: Red line
  - Moderate: Dashed blue line
  - High: Dotted green line
  - Time (s) range: 0 to 20

- **Δθ (deg)**
  - Low: Red line
  - Moderate: Dashed blue line
  - High: Dotted green line
  - Time (s) range: 0 to 20

- **Δδ_e (deg)**
  - Low: Red line
  - Moderate: Dashed blue line
  - High: Dotted green line
  - Time (s) range: 0 to 20

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Simulation Results

- Randomized initial conditions

- Noise/wind reconstruction
Numerical Results

- **Comparison with certainty equivalent MPC**

![Graph showing comparison between AD-SMPC and CE-MPC](image)

- **Cost comparison**

<table>
<thead>
<tr>
<th>Method</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>AD-SMPC</td>
<td>$3.23 \times 10^6$</td>
</tr>
<tr>
<td>CE-MPC</td>
<td>$3.47 \times 10^6$</td>
</tr>
</tbody>
</table>
Conclusions and Future Work

Summary

- Stochastic MPC for aircraft glideslope recovery in gust
- Chance constrained affine-disturbance feedback MPC formulation
- Tractable, cost efficient solution compared to certainty equivalent MPC

Future directions

- Incomplete state information and measurement noise
- Inclusion of carrier burble components