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Halo Orbit Stationkeeping using Nonlinear MPC and Polynomial Optimization

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- Introduction and motivation
- Problem Formulation
  - Motion in restricted three body problem
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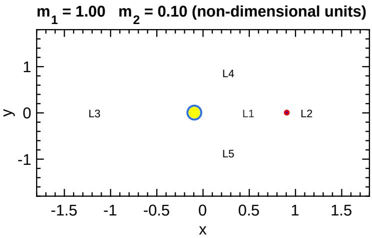

- Libration points: Ideal locations for human/robotic space exploration.
- Several successful past missions: ISEE-3, SOHO.
- Active station-keeping required.

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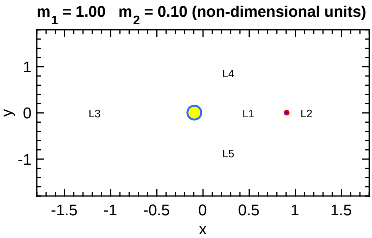

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- Common station-keeping approaches:
  - Discrete
    - Discrete LQR [Folta and Vaughn, 2004]
    - Chebyshev-Picard iterations [Bai and Junkins, 2012]
  - Continuous
    - Continuous LQR [Nazari et al., 2014]
    - Nonlinear optimization [Ulybyshev, 2015]
    - Linear MPC [Peng et al., 2017, Kalabić et al., 2015]
    - Nonlinear MPC [Li et al., 2015]
- Goal for this work: Globally optimal constrained receding horizon solution.

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Motion near libration points

- Equations of motion with origin at L1 libration point

$$\ddot{x} = 2\dot{y} + (1-\gamma_L) + x - \frac{(1-\mu)}{r_1^3}(1-\gamma_L+x) + \frac{\mu}{r_2^3}(\gamma_L - x) - \mu$$

$$\ddot{y} = -2\dot{x} + y - \frac{y(1-\mu)}{r_1^3} - \frac{y\mu}{r_2^3}$$

$$\ddot{z} = -(1-\mu)\frac{z}{r_1^3} - \mu\frac{z}{r_2^3}$$

- γ_L: Distance between L_1 and primary

Legendre Polynomial Approximation

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- Equations of motion in terms of legendre polynomials

$$\ddot{x} - 2\dot{y} - (1 + 2c_2)x = \frac{\partial}{\partial x} \sum_{n \geq 3} c_n \rho^n P_n(\frac{x}{\rho}) \quad (1)$$

$$\ddot{y} + 2\dot{x} + (c_2 - 1)y = \frac{\partial}{\partial y} \sum_{n \geq 3} c_n \rho^n P_n(\frac{x}{\rho}) \quad (2)$$

$$\ddot{z} + c_2 z = \frac{\partial}{\partial z} \sum_{n \geq 3} c_n \rho^n P_n(\frac{x}{\rho}) \quad (3)$$

- $c_n = \gamma_L^{-3}(\mu + (-1)^n(1 - \mu)(\frac{\gamma_L}{1-\gamma_L})^{n+1})$

Polynomial Approximations

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- Richardson's third order approximation

$$
\begin{aligned}
\ddot{x} - 2\dot{y} - (1+2c_2)x &= \frac{3}{2}c_3(2x^2 - y^2 - z^2) \\
&\quad + 2c_4x(2x^2 - 3y^2 - 3z^2) + \mathcal{O}(4) \\
\ddot{y} + 2\dot{x} + (c_2 - 1)y &= -3c_3xy - \frac{3}{2}c_4(4x^2 - y^2 - z^2)y \\
&\quad + \mathcal{O}(4) \\
\ddot{z} + c_2z &= -3c_3xz - \frac{3}{2}c_4z(4x^2 - y^2 - z^2) \\
&\quad + \mathcal{O}(4)
\end{aligned}
$$

- Analytic periodic solution based on Lindstedt-Poincarè perturbation method

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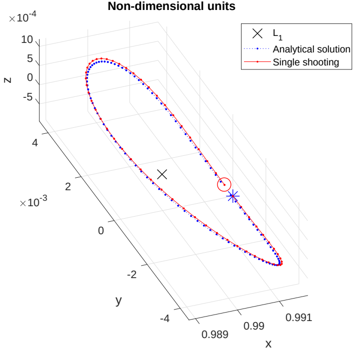
Polynomial Approximations

- Analytical methods: qualitatively insightful, but insufficient for dynamical analysis.
- Combined with differential correction for trajectory refinement.



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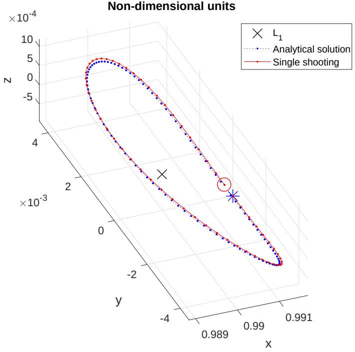

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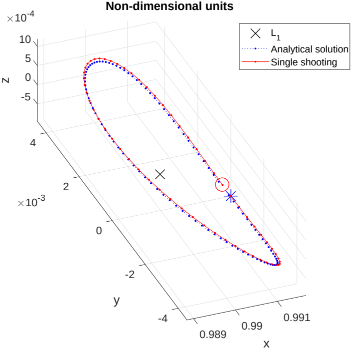

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Polynomial Approximations

- Taylor expansions considered in this study.
- Numerically more accurate compared to Legendre expansions.
- Second order Taylor expansion model.

$$\ddot{x} = 2\dot{y} + \left(\frac{3\mu}{2\gamma_L^4} - \frac{3\mu - 1}{2(\gamma_L - 1)^4}\right)\left(2x^2 - y^2 - z^2\right) + \left(\frac{2\mu}{\gamma_L^3} - \frac{2\mu - 1}{(\gamma_L - 1)^3}\right)x - \mu - \gamma_L + \frac{\mu}{\gamma_L^2} - \frac{\mu - 1}{(\gamma_L - 1)^2} + 1$$

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&+ \left(\frac{2\mu}{\gamma_L^3} - \frac{2\mu - 1}{(\gamma_L - 1)^3}\right)x - \mu - \gamma_L \\
&+ \frac{\mu}{\gamma_L^2} - \frac{\mu - 1}{(\gamma_L - 1)^2} + 1
\end{aligned}
$$

Polynomial Approximations

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$$\ddot{y} = -2\dot{x} - \left(\frac{3\mu}{\gamma_L^3} - \frac{3\mu - 1}{(\gamma_L - 1)^3}\right)xy + \left(1 - \frac{\mu}{\gamma_L^3} + \frac{\mu - 1}{(\gamma_L - 1)^3}\right)y$$

$$\ddot{z} = -\left(\frac{3\mu}{\gamma_L^3} - \frac{3\mu - 1}{(\gamma_L - 1)^3}\right)xz + \left(\frac{\mu - 1}{(\gamma_L - 1)^3} - \frac{\mu}{\gamma_L^3}\right)z$$

- Approximate polynomial model to be used for polynomial optimization based MPC.
- General methodology, can be applied to similar dynamical regimes.

Polynomial Approximations

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Polynomial Optimization

- Polynomial: finite combination of monomials

$$p(x) = \sum_{\alpha} c_\alpha x^\alpha = \sum_{\alpha} c_\alpha x_1^{\alpha_1} \dots x_n^{\alpha_n}, \quad c_\alpha \in \mathbb{R}$$

- Consider optimization problem

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & f(x) \\ \text{subject to} \quad & x \in \mathcal{K} \end{aligned}$$

  - $\mathcal{K} := \{x : g_j(x) <= 0, j = 1, 2, 3..\}$

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Polynomial Optimization

Definition

A polynomial is denoted as sum of squares (SOS) if it can be represented as

$$f(x) = \sum_{j \in \mathcal{J}} \left(g_j(x)\right)^2 \tag{4}$$

Alternatively, $f(x)$ with degree $2d$ and in n variables is SOS if

$$p(x) = z^{\top} Q z \tag{5}$$

where $Q \succ 0$ and $z = [1, x_1, x_2 \ldots x_n, x_1 x_2 \ldots x_n^d]$ is the vector of monomials upto degree d.

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Polynomial Optimization

- Non-convex and NP complete.
- Using lifted variables, expressed as with lifted variables as

$$\begin{aligned} \underset{x\in\mathcal{K},\lambda\in\mathbb{R}}{\text{maximize}} \quad & \lambda \\ \text{subject to} \quad & f(x)-\lambda \geq 0 \end{aligned} \tag{6}$$

- Approach: Relax problem by replacing non-negativity with positivity.

Polynomial Optimization

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Sum-of-Squares and Positivity Certificates

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- Weak (relaxed) form of polynomial optimization in terms of positivity (in particular sum-of-squares (SOS))

$$\underset{x\in\mathcal{K},\lambda\in\mathbb{R}}{\text{maximize}} \quad \lambda$$

$$\text{subject to} \quad f(x) - \lambda = s_0(x) + \sum_{j=1}^{m} s_j(x) g_j(x)$$

- SOS formulation comes from Putinar's positivstellensatz.

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Lemma (Putinar's positivstellensatz)

Define the quadratic module generated by g_j as Q_g

$$Q(g) := \sigma_0 + \sum_{j=1}^{m} \sigma_j g_j$$

where $(\sigma)_{j=0}^{m}$ are SOS. Assume there exists $u \in Q_g$ such that the level set $\{x \in \mathbb{R}^n : u(x) \geq 0\}$ is compact. If $f(x) > 0$ on $\mathcal{K}$, then $f(x) \in Q_g$ (for some SOS polynomials $(s(x))_{j=0}^{m}$).

$$f(x) = s_0 + \sum_{j=0}^{m} s_j g_j$$

Sum-of-Squares and Positivity Certificates

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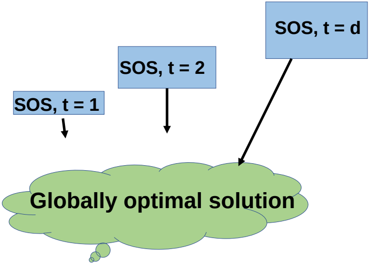
- Result: Semidefinite program (SDP), solvable using interior point methods.
- Relaxed SDP solution: P_{SOS} (lower bound).
- Bounds can be strengthened by increasing t.

$$P_{SOS}(t) \leq P_{SOS}(t+1) \leq P^*$$



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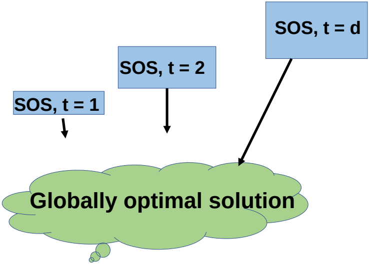

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Positivity Certificates and Dual Moment Relaxations

- Alternate positivity certificates exist: Krivine-Vasilescu-Handelman (LP), Schmüdgen (SOS)
- Dual framework: Moment SOS approach

$$\begin{aligned} \underset{y}{\text{maximize}} \quad & c^{\mathsf{T}} y \\ \text{subject to} \quad & M_t(y) \succeq 0 \\ & M_{t_j}(g_j y) \succeq 0, j = 1, 2..m \\ & y_0 = 1 \end{aligned}$$

- Solve hierarchy of SDPs with $\Uparrow t$ (Lasserre Hierarchy).
- Monotonic convergence to global optimum [Lasserre, 2001]

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Model Predictive Control

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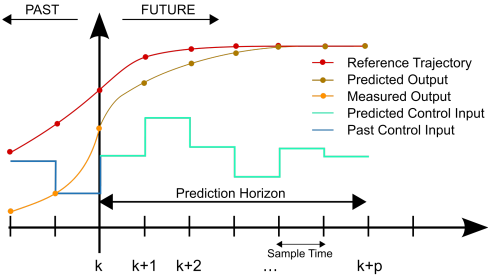
- Repeated solution of a constrained open-loop optimal control problem.



- For non-convex problems: locally optimal solutions.

Model Predictive Control

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- Consider $f(.,.)$ as polynomial vector field.
- Assume $\mathcal{X}$ and $\mathcal{U}$ as basic real semi-algebraic sets.
- In compact form

$$\zeta = [x_{k+1|k}, x_{k+2|k}, \ldots . x_{k+N|k}]^{\mathrm{T}}$$

- Using a recursive relation, $\nu = [u_{k|k}, \ldots . u_{k+N-1|k}]^{\mathrm{T}}$ and x_k

$$\zeta = F(\nu, x_k)$$

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Model Predictive Control

Theorem

[Raff et al., 2006] The finite horizon optimal control problem can be formulated as a polynomial optimization problem of the form

$$\underset{\nu \in \mathcal{K}}{\text{minimize}} \quad p_0(\nu)$$

for discrete time polynomial systems, if $\mathcal{K} = \{\nu \in \mathbb{R}^{m.N} : p_i(\nu) \geq 0, i = 1, 2 ... 2(n+m)N+1\}$, *is a compact set described by* $p_i(\nu) \in \mathbb{R}[\nu], i = 1, 2 ... 2(n+m)N+1.$

- Approach: Repeatedly solve polynomial optimization in receding horizon manner using moment-SOS approach.

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- MPC problem for trajectory tracking

$$\begin{aligned}
\underset{u}{\text{minimize}} \quad & \sum_{i=1}^{N_p-1} e_{i|k}^T Q e_{i|k} + u_{i|k}^T R u_{i|k} + e_{N_p|k}^T P e_{N_p|k} \\
\text{subject to} \quad & x_{i+1|k} = f(x_{i|k}, u_{i|k}) \\
& u_{i|k} \in \mathcal{U} \\
& x_{i|k} \in \mathcal{X} \\
& x_{k+N} \in \mathcal{X}_f, \quad i = k, k+1 \ldots k+N-1
\end{aligned}$$

  - Tracking error: $e_{i|k} = x_{i|k} - x_{i|k}^d$.
  - $Q \succeq 0$, $R \succ 0$.

Sun-Earth CRTBP Halo Orbit Tracking

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- Sun-Earth L1 halo orbit station-keeping.
- Large insertion error: $\approx 40,000$ km in x direction.
- Polynomial MPC parameters

Parameter	Value
Q	$\mathrm{diag}\left(\begin{bmatrix}10^6 & 10^6 & 10^6 & 1 & 1 & 1\end{bmatrix}\right)$
P	Discrete algebraic Riccati solution
R	$\mathrm{diag}\left(\begin{bmatrix}10^3 & 10^3 & 10^3\end{bmatrix}\right)$
N_p	15
N	60

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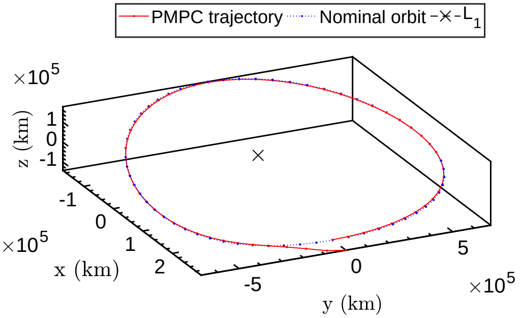
Sun-Earth CRTBP Halo Orbit Tracking

- Three-dimensional polynomial MPC trajectory
- Coordinate frame centered at L1



Sun-Earth CRTBP Halo Orbit Tracking

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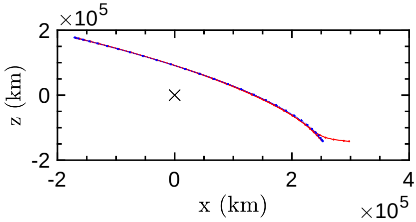
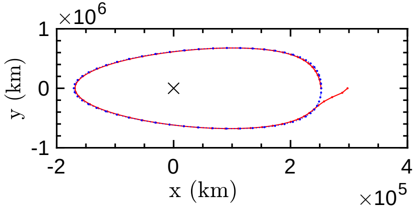
- Trajectory projection in XY, XZ plane





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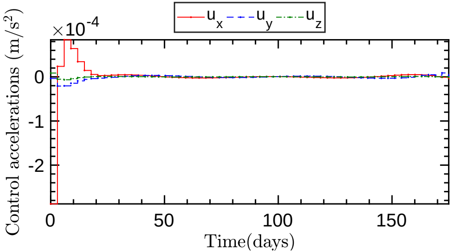

Scheme	ΔV (ms^{-2})	Solver
PMPC (global)	7.94×10^{-4}	Gloptipoly with Mosek
Nominal NMPC	6.81×10^{-4}	IPOPT
PMPC (local)	9.02×10^{-4}	IPOPT
LQR	7.01×10^{-4}	NA
LMPC	9.78×10^{-4}	Gurobi

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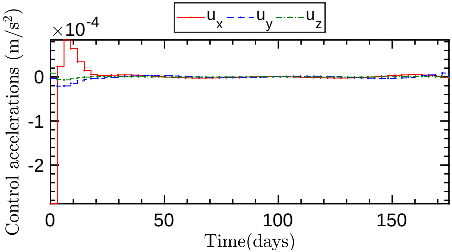

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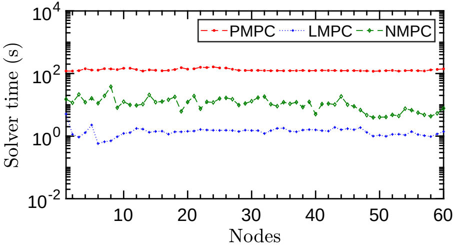

N_p	2	6	10
PMPC ΔV	0.001	8.45×10^{-4}	8.11×10^{-4}
NMPC ΔV	DNC	6.94×10^{-4}	6.87×10^{-4}
LMPC ΔV	DNC	0.001	0.0012
LQR ΔV	7.01×10^{-4}	7.01×10^{-4}	7.01×10^{-4}

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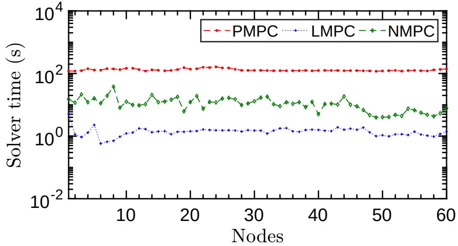

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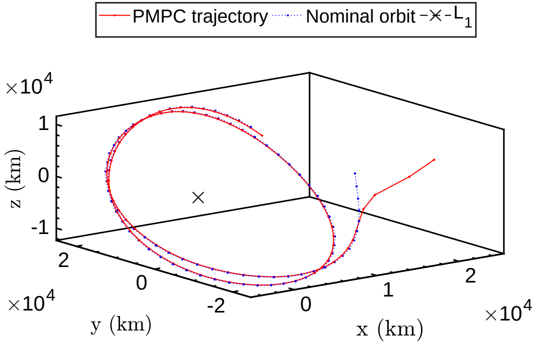
Earth-Moon CRTBP Lissajous Orbit Tracking

- Nominal orbit corrected by multiple shooting.
- Insertion error: 9500 km in x-direction



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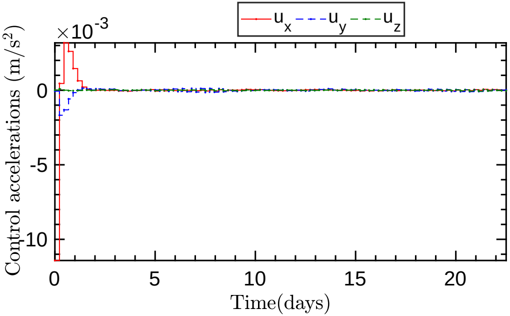
Earth-Moon CRTBP Lissajous Orbit Tracking

- Horizon length: 100, 22 days
- Prediction horizon: 10, 5.45 hours
- Control accelerations



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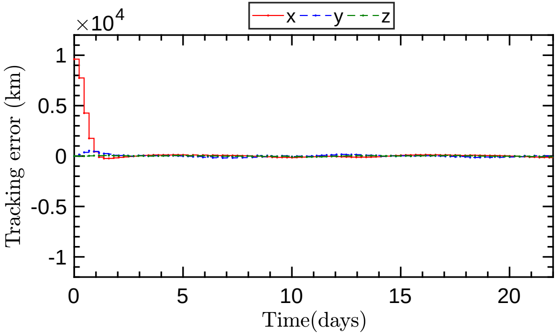
- Tracking errors



- Tracking convergence in approximately 3 days.

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Earth-Moon CRTBP Lissajous Orbit Tracking

N_p	2	6	10	15
PMPC ΔV	0.059	0.03212	0.03206	0.0318
NMPC ΔV	DNC	0.03219	0.03212	0.032
LMPC ΔV	0.117	0.0475	0.047	0.0465
LQR ΔV	0.12	0.12	0.12	0.12

- Overall, PMPC outperforms LQR, LMPC.
- Similar performance as NMPC but no initialization or warm start required.

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Conclusions and Future Work

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  - Polynomial optimization based MPC proposed.
    - Globally optimal solutions
    - No initial guess needed.
    - Limited by prediction horizon length.
- Future work
  - Consider full ephemeris model.
  - Robust MPC based techniques to ensure controller performance in presence of uncertainty.

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